GU J Sci 30(2): 205-218 (2017)

Gazi University

**Journal of Science**

http://dergipark.gov.tr/gujs

# Estimation of population variance using quartiles in absence and presence of non-response

Usman SHAHZAD[1], Muhammad HANIF[1], Nursel KOYUNCU[2], A. V. Garcia LUENGO[3]

[1]*Department of Mathematics and Statisitcs, PMAS-Arid Agriculture University, Rawalpindi, Pakistan.*
[2]*Department of Statisitcs, University of Hacettepe, Ankara, Turkey.*
[3]*Department of Mathematics, University of Almeria, Spain.*

**Article Info**

*Received: 22/11/2016*
*Revised: 05/03/2017*
*Accepted: 04/04/2017*

**Keywords**

*Mean Square Error*
*Non-response*
*Simple Random Sampling*
*Quartiles*

**Abstract**

In this study, we have addressed the issue of estimation of population variance utilizing quartiles and some of their functions of an auxiliary variable in absence and presence of non-response. A class of ratio type estimators have been suggested with their MSEs' in simple random sampling. The suggested estimators have been compared with the reviewed estimators. Further, an empirical illustration is carried out to support the theoretical findings.

## 1. INTRODUCTION

In real life there are various situations where the estimation of population variance assumes importance such as industry, agriculture, biology and medical studies have been facing the problem in evaluating the finite population variance. For instance a doctor needs a full comprehension of variation in the degree of human circulatory strain, body temperature and heartbeat rate for adequate remedy. Similarly, an agriculturalist requires sufficient information of climatic variety to devise suitable arrangement for developing his product. A reasonable comprehension of variability is essential for better results in different fields of life. For these reasons various authors such as Isaki (1983), Singh et al. (1988), Upadhyaya and Singh (1999), Kadilar and Cingi (2006a), Kadilar and Cingi (2006b), Solanki et al. (2015) and Sinha and Kumar (2015) have paid their attention towards the enhanced estimation of population variance $S_y^2$ of the study variable Y. Non-response is also important issue in literature. Recently, Riaz et al. (2014) and Singh et al. (2016) have proposed a generalized class of estimators for population mean under different sampling designs in presence of non-response. In this study our main aim is the estimation of finite population variance of Y by enhancing the previous estimators utilizing information of an auxiliary variable such as quartiles and some of their functions. Further, the study is also extended for non-response problem.

## 2. PRELIMINARIES AND EXISTING ESTIMATORS

To find the mean square error of the proposed and existing estimators, let us define:

$$\delta_o = \frac{s_y^2 - S_y^2}{S_y^2},\ E(\delta_o^2) = \lambda\beta_2(y)^{\diamond} = u_{20}, E(\delta_o) = E(\delta_1) = 0,$$

*Corresponding author, e-mail: usman.stat@yahoo.com, mhpuno@hotmail.com, nkoyuncu@hacettepe.edu.tr, amgarcia@ual.es

$\boldsymbol{\delta_1 = \frac{s_x^2 - S_x^2}{S_x^2}},\ \boldsymbol{E(\delta_o^2) = \lambda\beta_2(x)^\diamond = u_{02}, E(\delta_o\delta_1) = \lambda\eta_{22} = u_{11}},$

$\boldsymbol{\beta_2(y)^\diamond = \beta_2(y) - 1 = \frac{\mu_{40}}{\mu_{20}^2} - 1 = 0, \beta_2(x)^\diamond = \beta_2(x) - 1 = \frac{\mu_{04}}{\mu_{02}^2} - 1},$

$\boldsymbol{\eta_{22}^\diamond = \eta_{22} - 1, \eta_{22} = \frac{\mu_{22}}{\mu_{20}\mu_{02}}}, \rho^\diamond = \frac{\eta_{22}^\diamond}{\sqrt{\beta_2(x)^\diamond\beta_2(y)^\diamond}},$

where

$\lambda = \left(\frac{1}{n} - \frac{1}{N}\right), \mu_{rk} = N^{-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^r(X_i - \bar{X})^k$ and $u_{rk} = E\frac{\left\{(s_y^2 - S_y^2)^r(s_x^2 - S_x^2)^k\right\}}{(s_y^2)^r(s_x^2)^k}$.

The usual variance estimator is

$$\hat{T}_o = s_y^2 \quad \textbf{(2.1)}$$

$$Var(\hat{T}_o) = S_y^4 u_{20} \quad \textbf{(2.2)}$$

Singh et al. (1973) introduced the following estimator for the estimation of $s_y^2$

$$\hat{T}_{se} = k_{se}s_y^2, \quad \textbf{(2.3)}$$

where $k_{se}$ be the chosen constant.

The MSE of $\hat{T}_{se}$ is given by

$$MSE(\hat{T}_{se}) = S_y^4\left[(k_{se} - 1)^2 + k_{se}^2 u_{20}\right], \quad \textbf{(2.4)}$$

The MSE of $\hat{T}_{se}$ is minimum for $k_{se}^{opt} = \left[\frac{1}{1+u_{20}}\right]$ as given by

$$MSE_{min}(\hat{T}_{se}) = S_y^4\left[\frac{u_{20}}{1+u_{20}}\right]. \quad \textbf{(2.5)}$$

Isaki (1983) introduced the following ratio type estimator for the estimation of $s_y^2$ as

$$\hat{T}_{isaki} = s_y^2\frac{S_x^2}{s_x^2}. \quad \textbf{(2.6)}$$

The mean square error of $\hat{T}_{isaki}$ is given by

$$MSE(\hat{T}_{isaki}) = S_y^4[u_{20} + u_{02} - 2u_{11}]. \quad \textbf{(2.7)}$$

Upahyaya and Singh (1999) introduced the following ratio type estimator for the estimation of $s_y^2$ as

$$\hat{T}_{us} = s_y^2\left[\frac{S_x^2+\beta_2(x)}{s_x^2+\beta_2(x)}\right]. \tag{2.8}$$

The mean square error of $\hat{T}_{us}$ is given by

$$MSE(\hat{T}_{us}) = S_y^4\left[u_{20}+\theta_{us}^2 u_{02}-2\theta_{us}u_{11}\right], \tag{2.9}$$

where $\theta_{us} = \frac{S_x^2}{S_x^2+\beta_2(x)}$ .

Kadilar and Cingi (2006) introduced the class of estimators for the estimation of $s_y^2$ as

$$\hat{T}_{kc1} = s_y^2\left[\frac{S_x^2-C_x}{s_x^2-C_x}\right], \tag{2.10}$$

$$\hat{T}_{kc2} = s_y^2\left[\frac{S_x^2-\beta_2(x)}{s_x^2-\beta_2(x)}\right], \tag{2.11}$$

$$\hat{T}_{kc3} = s_y^2\left[\frac{\beta_2(x)\, S_x^2-C_x}{\beta_2(x)\, s_x^2-C_x}\right], \tag{2.12}$$

$$\hat{T}_{kc4} = s_y^2\left[\frac{C_x\, S_x^2-\beta_2(x)}{C_x\, s_x^2-\beta_2(x)}\right]. \tag{2.13}$$

The mean square errors of these estimators are given below

$$MSE(\hat{T}_{kc1}) = S_y^4\left[u_{20}+\theta_{kc1}^2 u_{02}-2\theta_{kc1}u_{11}\right], \tag{2.14}$$

$$MSE(\hat{T}_{kc2}) = S_y^4\left[u_{20}+\theta_{kc2}^2 u_{02}-2\theta_{kc2}u_{11}\right], \tag{2.15}$$

$$MSE(\hat{T}_{kc3}) = S_y^4\left[u_{20}+\theta_{kc3}^2 u_{02}-2\theta_{kc3}u_{11}\right], \tag{2.16}$$

$$MSE(\hat{T}_{kc4}) = S_y^4\left[u_{20}+\theta_{kc4}^2 u_{02}-2\theta_{kc4}u_{11}\right], \tag{2.17}$$

where $\theta_{kc1} = \frac{S_x^2}{S_x^2-C_x}$, $\theta_{kc2} = \frac{S_x^2}{S_x^2-\beta_2(x)}$, $\theta_{kc3} = \frac{\beta_2(x)S_x^2}{\beta_2(x)S_x^2-C_x}$, $\theta_{kc4} = \frac{C_xS_x^2}{C_xS_x^2-\beta_2(x)}$ .

Singh et al. (2015) introduced the class of estimators for the estimation of $s_y^2$ as

$$\hat{T}_{s1} = s_y^2\left[\frac{S_x^2+\alpha Q_1^2}{s_x^2-\alpha Q_1^2}\right], \tag{2.18}$$

$$\hat{T}_{s2} = s_y^2\left[\frac{S_x^2+\alpha Q_2^2}{s_x^2-\alpha Q_2^2}\right], \tag{2.19}$$

$$\hat{T}_{s3} = s_y^2\left[\frac{S_x^2+\alpha Q_3^2}{s_x^2-\alpha Q_3^2}\right], \tag{2.20}$$

$$\hat{T}_{s4} = s_y^2\left[\frac{S_x^2+\alpha Q_r^2}{s_x^2-\alpha Q_r^2}\right], \tag{2.21}$$

$$\hat{T}_{s5} = s_y^2\left[\frac{S_x^2+\alpha Q_d^2}{s_x^2-\alpha Q_d^2}\right], \tag{2.22}$$

$$\hat{T}_{s6} = s_y^2\left[\frac{S_x^2+\alpha Q_a^2}{s_x^2-\alpha Q_a^2}\right], \tag{2.23}$$

Where

$Q_r = \{Q_3 - Q_1\}$, $Q_d = \frac{\{Q_3-Q_1\}}{2}$, , $Q_a = \frac{\{Q_3+Q_1\}}{2}$.

For ease in calculation we consider $\alpha=1$. The mean square errors of these estimators are given below

$$MSE(\hat{T}_{s1}) = S_y^4[u_{20} + \theta_{s1}^2 u_{02} - 2\theta_{s1}u_{11}], \tag{2.24}$$

$$MSE(\hat{T}_{s2}) = S_y^4[u_{20} + \theta_{s2}^2 u_{02} - 2\theta_{s2}u_{11}], \tag{2.25}$$

$$MSE(\hat{T}_{s3}) = S_y^4[u_{20} + \theta_{s3}^2 u_{02} - 2\theta_{s3}u_{11}], \tag{2.26}$$

$$MSE(\hat{T}_{s4}) = S_y^4[u_{20} + \theta_{s4}^2 u_{02} - 2\theta_{s4}u_{11}], \tag{2.27}$$

$$MSE(\hat{T}_{s5}) = S_y^4[u_{20} + \theta_{s5}^2 u_{02} - 2\theta_{s5}u_{11}], \tag{2.28}$$

$$MSE(\hat{T}_{s6}) = S_y^4[u_{20} + \theta_{s6}^2 u_{02} - 2\theta_{s6}u_{11}]. \tag{2.29}$$

where $\theta_{s1} = \frac{S_x^2}{S_x^2+Q_1^2}$, $\theta_{s2} = \frac{S_x^2}{S_x^2+Q_2^2}$, $\theta_{s3} = \frac{S_x^2}{S_x^2+Q_3^2}$, $\theta_{s4} = \frac{S_x^2}{S_x^2+Q_r^2}$, $\theta_{s5} = \frac{S_x^2}{S_x^2+Q_d^2}$, $\theta_{s6} = \frac{S_x^2}{S_x^2+Q_a^2}$.

## 3. SUGGESTED CLASS OF ESTIMATORS

By adapting Singh et al. (1973) and Singh et al. (2015), we suggest the following generalized class of estimators' as

$$\hat{T}_{pi} = k_{pi} s_y^2 \left[\frac{S_x^2 + \Omega_i^2}{s_x^2 + \Omega_i^2}\right], \tag{3.30}$$

Where $\Omega_i$ be any known population characteristic.

Suppose quartiles and there functions are known than some of the family members of $\hat{T}_{pi}$ are as follows

$$\hat{T}_{p1} = k_{p1} s_y^2 \left[\frac{S_x^2 + \alpha Q_1^2}{s_x^2 - \alpha Q_1^2}\right], \tag{3.31}$$

$$\hat{T}_{p2} = k_{p2} s_y^2 \left[\frac{S_x^2 + \alpha Q_2^2}{s_x^2 - \alpha Q_2^2}\right], \tag{3.32}$$

$$\hat{T}_{p3} = k_{p3} s_y^2 \left[\frac{S_x^2 + \alpha Q_3^2}{s_x^2 - \alpha Q_3^2}\right], \tag{3.33}$$

$$\hat{T}_{p4} = k_{p4} s_y^2 \left[\frac{S_x^2 + \alpha Q_r^2}{s_x^2 - \alpha Q_r^2}\right], \tag{3.34}$$

$$\hat{T}_{p5} = k_{p5} s_y^2 \left[\frac{S_x^2 + \alpha Q_d^2}{s_x^2 - \alpha Q_d^2}\right], \tag{3.35}$$

$$\hat{T}_{p6} = k_{p6} s_y^2 \left[\frac{S_x^2 + \alpha Q_a^2}{s_x^2 - \alpha Q_a^2}\right]. \tag{3.36}$$

The mean square error of the proposed class can be written as follows

$$MSE(\hat{T}_{pi}) = S_y^4\left[1 + k_{pi}^2\left(1 + u_{20} + 3\theta_{pi}^2 u_{02} - 4\theta_{pi} u_{11}\right) - 2k_{pi}\left(1 + \theta_{pi}^2 u_{02} - \theta_{pi} u_{11}\right)\right], \tag{3.37}$$

where $\theta_{pi} = \theta_{si}$ for (i=1,2,…,6).

The MSE of $\hat{T}_{pi}$ for (i=1,2,…,6) is minimum for $k_{pi}^{opt} = \frac{1+\theta_{pi}^2 u_{02} - \theta_{pi} u_{11}}{1+u_{20}+3\theta_{pi}^2 u_{02} - 4\theta_{pi} u_{11}}$ as given by

$$MSE_{min}(\hat{T}_{pi}) = S_y^4\left[1 - \frac{\left(1+\theta_{pi}^2 u_{02} - \theta_{pi} u_{11}\right)^2}{1 + u_{20} + 3\theta_{pi}^2 u_{02} - 4\theta_{pi} u_{11}}\right]. \quad \textbf{(3.38)}$$

## 4. NON-RESPONSE

Hansen and Hurwitz (1946) sub-sampling scheme is the most widely used technique for the non-response problem. In this scheme, let $n_1$ be the responding units out of *n* and remaining $n_2 = n - n_1$ units are taken as non-respondents from the whole population $U_N$ (say). Now a sub sample of size $n_g = \frac{n_2}{l}$ is selected by SRSWOR from $n_2$ non-respondent units with the inverse sampling rate *l* i.e ($l > 1$). Note that all $n_g$ units fully respond on second call. The population is said to be distributed into 2 groups namely $U_{N1}$ and $U_{N2}$ of sizes $N_1$ and $N_2$, Further $U_{N1}$ is a response group that would give response on the first call and $U_{N2}$ is non-response group which could respond on the second call. Obviously $U_{N1}$ and $U_{N2}$ are non-overlapping and unknown quantities.

Recently, Sinha and Kumar (2015) find the following unbiased estimator for handling the non-response issue in the estimation of population variance

$$\hat{T}'_o = s'^2_y = \frac{1}{n-1}\left(\sum_{U_{n1}} y_i^2 + l \sum_{U_{n2(ng)}} y_i^2 - \hat{\bar{y}}'^2\right), \quad \textbf{(4.39)}$$

where $\hat{\bar{y}}'$ is a Hansen and Hurwitz (1946) unbiased estimator for the estimation of $\bar{Y}$ in case of non-response.

$$Var(\hat{T}'_o) = S_y^4 u_{20} + wS_{y(2)}^4\left(\beta_2(y_{(2)}) - 1\right) = S_y^4 u_{20} + wS_{y(2)}^4 \beta_2(y_{(2)})^{\diamond}, \quad \textbf{(4.40)}$$

Where $w = \frac{N_2(l-1)}{nN}$.

Singh et al. (1973) estimator for non-response is given by

$$\hat{T}'_{se} = k_{se} s'^2_y, \quad \textbf{(4.41)}$$

The minimum MSE of $\hat{T}'_{se}$ is given by

$$MSE_{min}(\hat{T}'_{se}) = S_y^4\left[\frac{u'_{20}}{1+u'_{20}}\right], \quad \textbf{(4.42)}$$

where

$$u'_{20} = \frac{Var(\hat{T}'_o)}{S_y^4} = u_{20} + \frac{wS_{y(2)}^4 \beta_2(y)^{\diamond}}{S_y^4}.$$

Isaki (1983) ratio type estimator for non-response is given by

$$\hat{T}'_{isaki} = s_y'^2 \frac{S_x^2}{s_x^2}. \quad \textbf{(4.43)}$$

The mean square error of $\hat{T}'_{isaki}$ is given by

$$MSE(\hat{T}'_{isaki}) = S_y^4[u'_{20} + u_{02} - 2u_{11}]. \quad \textbf{(4.44)}$$

Upadhyaya and Singh (1999) ratio type estimator for non-response is given by

$$\hat{T}'_{us} = s_y'^2 \left[\frac{S_x^2 + \beta_2(x)}{s_x^2 + \beta_2(x)}\right]. \quad \textbf{(4.45)}$$

The mean square error of $\hat{T}'_{us}$ is given by

$$MSE(\hat{T}'_{us}) = S_y^4[u'_{20} + \theta_{us}^2 u_{02} - 2\theta_{us} u_{11}]. \quad \textbf{(4.46)}$$

Kadilar and Cingi (2006) estimators for non-response can be introduced as follows

$$\hat{T}'_{kc1} = s_y'^2 \left[\frac{S_x^2 - C_x}{s_x^2 - C_x}\right], \quad \textbf{(4.47)}$$

$$\hat{T}'_{kc2} = s_y'^2 \left[\frac{S_x^2 - \beta_2(x)}{s_x^2 - \beta_2(x)}\right], \quad \textbf{(4.48)}$$

$$\hat{T}'_{kc3} = s_y'^2 \left[\frac{\beta_2(x)\, S_x^2 - C_x}{\beta_2(x)\, s_x^2 - C_x}\right], \quad \textbf{(4.49)}$$

$$\hat{T}'_{kc4} = s_y'^2 \left[\frac{C_x\, S_x^2 - \beta_2(x)}{C_x\, s_x^2 - \beta_2(x)}\right]. \quad \textbf{(4.50)}$$

The mean square errors of these estimators are given below

$$MSE(\hat{T}'_{kc1}) = S_y^4[u'_{20} + \theta_{kc1}^2 u_{02} - 2\theta_{kc1} u_{11}], \quad \textbf{(4.51)}$$

$$MSE(\hat{T}'_{kc2}) = S_y^4[u'_{20} + \theta_{kc2}^2 u_{02} - 2\theta_{kc2} u_{11}], \quad \textbf{(4.52)}$$

$$MSE(\hat{T}'_{kc3}) = S_y^4[u'_{20} + \theta_{kc3}^2 u_{02} - 2\theta_{kc3} u_{11}], \quad \textbf{(4.53)}$$

$$MSE(\hat{T}'_{kc4}) = S_y^4[u'_{20} + \theta_{kc4}^2 u_{02} - 2\theta_{kc4} u_{11}], \quad \textbf{(4.54)}$$

Singh et al. (2015) estimators for non-response can be introduced as follows

$$\hat{T}'_{s1} = s_y'^2 \left[\frac{S_x^2 + \alpha Q_1^2}{s_x^2 - \alpha Q_1^2}\right], \tag{4.55}$$

$$\hat{T}'_{s2} = s_y'^2 \left[\frac{S_x^2 + \alpha Q_2^2}{s_x^2 - \alpha Q_2^2}\right], \tag{4.56}$$

$$\hat{T}'_{s3} = s_y'^2 \left[\frac{S_x^2 + \alpha Q_3^2}{s_x^2 - \alpha Q_3^2}\right], \tag{4.57}$$

$$\hat{T}'_{s4} = s_y'^2 \left[\frac{S_x^2 + \alpha Q_r^2}{s_x^2 - \alpha Q_r^2}\right], \tag{4.58}$$

$$\hat{T}'_{s5} = s_y'^2 \left[\frac{S_x^2 + \alpha Q_d^2}{s_x^2 - \alpha Q_d^2}\right], \tag{4.59}$$

$$\hat{T}'_{s6} = s_y'^2 \left[\frac{S_x^2 + \alpha Q_a^2}{s_x^2 - \alpha Q_a^2}\right]. \tag{4.60}$$

$$MSE(\hat{T}'_{s1}) = S_y^4[u'_{20} + \theta_{s1}^2 u_{02} - 2\theta_{s1} u_{11}], \tag{4.61}$$

$$MSE(\hat{T}'_{s2}) = S_y^4[u'_{20} + \theta_{s2}^2 u_{02} - 2\theta_{s2} u_{11}], \tag{4.62}$$

$$MSE(\hat{T}'_{s3}) = S_y^4[u'_{20} + \theta_{s3}^2 u_{02} - 2\theta_{s3} u_{11}], \tag{4.63}$$

$$MSE(\hat{T}'_{s4}) = S_y^4[u'_{20} + \theta_{s4}^2 u_{02} - 2\theta_{s4} u_{11}], \tag{4.64}$$

$$MSE(\hat{T}'_{s5}) = S_y^4[u'_{20} + \theta_{s5}^2 u_{02} - 2\theta_{s5} u_{11}], \tag{4.65}$$

$$MSE(\hat{T}'_{s6}) = S_y^4[u'_{20} + \theta_{s6}^2 u_{02} - 2\theta_{s6} u_{11}]. \tag{4.66}$$

### 4.1. Suggested class of estimators under non-response

The Suggested class of estimators under non-response can be defined as follows

$$\hat{T}_{pi} = k_{pi} s_y'^2 \left[\frac{S_x^2 + \Omega_i^2}{s_x^2 + \Omega_i^2}\right], \tag{4.67}$$

Where $\boldsymbol{\Omega_i}$ be any known population characteristic.

Some of the family members of $\hat{T}'_{pi}$ are as follows

$$\hat{\boldsymbol{T}}'_{p1} = k_{p1} s_y'^2 \left[\frac{S_x^2 + \alpha Q_1^2}{s_x^2 - \alpha Q_1^2}\right], \qquad \textbf{(4.68)}$$

$$\hat{\boldsymbol{T}}'_{p2} = k_{p2} s_y'^2 \left[\frac{S_x^2 + \alpha Q_2^2}{s_x^2 - \alpha Q_2^2}\right], \qquad \textbf{(4.69)}$$

$$\hat{\boldsymbol{T}}'_{p3} = k_{p3} s_y'^2 \left[\frac{S_x^2 + \alpha Q_3^2}{s_x^2 - \alpha Q_3^2}\right], \qquad \textbf{(4.70)}$$

$$\hat{\boldsymbol{T}}'_{p4} = k_{p4} s_y'^2 \left[\frac{S_x^2 + \alpha Q_r^2}{s_x^2 - \alpha Q_r^2}\right], \qquad \textbf{(4.71)}$$

$$\hat{\boldsymbol{T}}'_{p5} = k_{p5} s_y'^2 \left[\frac{S_x^2 + \alpha Q_d^2}{s_x^2 - \alpha Q_d^2}\right], \qquad \textbf{(4.72)}$$

$$\hat{\boldsymbol{T}}'_{p6} = k_{p6} s_y'^2 \left[\frac{S_x^2 + \alpha Q_a^2}{s_x^2 - \alpha Q_a^2}\right]. \qquad \textbf{(4.73)}$$

The mean square error of the proposed class can be written as follows

$$MSE(\hat{\boldsymbol{T}}'_{pi}) = S_y^4\left[1 + k_{pi}^2\left(1 + u'_{20} + 3\theta_{pi}^2 u_{02} - 4\theta_{pi}u_{11}\right) - 2k_{pi}\left(1 + \theta_{pi}^2 u_{02} - \theta_{pi}u_{11}\right)\right]. \qquad \textbf{(4.74)}$$

The MSE of $\hat{\boldsymbol{T}}'_{pi}$ for (i=1,2,…,6) is minimum for $k_{pi}^{opt} = \frac{1+\theta_{pi}^2 u_{02} - \theta_{pi}u_{11}}{1+u'_{20}+3\theta_{pi}^2 u_{02} - 4\theta_{pi}u_{11}}$ as given by

$$MSE_{min}(\hat{\boldsymbol{T}}'_{pi}) = S_y^4\left[1 - \frac{\left(1 + \theta_{pi}^2 u_{02} - \theta_{pi}u_{11}\right)^2}{1 + u'_{20} + 3\theta_{pi}^2 u_{02} - 4\theta_{pi}u_{11}}\right]. \qquad \textbf{(4.75)}$$

## 5. EFFICIENCY COMPARISON

In current section, we find the efficiency conditions for the proposed class by looking at the minimum mean square error of the existing estimators in absence of non-response as

Observation (A):

$MSE_{min}(\hat{\boldsymbol{T}}_{pi}) < MSE(\hat{\boldsymbol{T}}_o)$,

if

$\left[1 - \frac{\left(1+\theta_{pi}^2 u_{02} - \theta_{pi}u_{11}\right)^2}{1+u_{20}+3\theta_{pi}^2 u_{02} - 4\theta_{pi}u_{11}}\right] - u_{20} < 0,$

Observation (B):

$\boldsymbol{MSE_{min}}(\hat{\boldsymbol{T}}_{\boldsymbol{pi}}) < \boldsymbol{MSE_{min}}(\hat{\boldsymbol{T}}_{\boldsymbol{se}})$,

if

$$\left[1-\frac{\left(1+\theta_{pi}^2 u_{02}-\theta_{pi}u_{11}\right)^2}{1+u_{20}+3\theta_{pi}^2 u_{02}-4\theta_{pi}u_{11}}\right]-\left[\frac{u_{20}}{1+u_{20}}\right]<0,$$

Observation (C):

$\boldsymbol{MSE_{min}}(\hat{\boldsymbol{T}}_{\boldsymbol{pi}}) < \boldsymbol{MSE}(\hat{\boldsymbol{T}}_{\boldsymbol{isaki}})$,

if

$$\left[1-\frac{\left(1+\theta_{pi}^2 u_{02}-\theta_{pi}u_{11}\right)^2}{1+u_{20}+3\theta_{pi}^2 u_{02}-4\theta_{pi}u_{11}}\right]-[u_{20}+u_{02}-2u_{11}]<0,$$

Observation (D):

$\boldsymbol{MSE_{min}}(\hat{\boldsymbol{T}}_{\boldsymbol{pi}}) < \boldsymbol{MSE}(\hat{\boldsymbol{T}}_{\boldsymbol{us}})$,

if

$$\left[1-\frac{\left(1+\theta_{pi}^2 u_{02}-\theta_{pi}u_{11}\right)^2}{1+u_{20}+3\theta_{pi}^2 u_{02}-4\theta_{pi}u_{11}}\right]-[u_{20}+\theta_{us}^2 u_{02}-2\theta_{us}u_{11}]<0,$$

Observation (E):

$\boldsymbol{MSE_{min}}(\hat{\boldsymbol{T}}_{\boldsymbol{pi}}) < \boldsymbol{MSE}(\hat{\boldsymbol{T}}_{\boldsymbol{kci}})$, for (i=1,2,3,4)

if

$$\left[1-\frac{\left(1+\theta_{pi}^2 u_{02}-\theta_{pi}u_{11}\right)^2}{1+u_{20}+3\theta_{pi}^2 u_{02}-4\theta_{pi}u_{11}}\right]-\left[u_{20}+\theta_{kci}^2 u_{02}-2\theta_{kci}u_{11}\right]<0,$$

Observation (F):

$\boldsymbol{MSE_{min}}(\hat{\boldsymbol{T}}_{\boldsymbol{pi}}) < \boldsymbol{MSE}(\hat{\boldsymbol{T}}_{\boldsymbol{si}})$, for (i=1,2,3,4,5,6)

if

$$\left[1-\frac{\left(1+\theta_{pi}^2 u_{02}-\theta_{pi}u_{11}\right)^2}{1+u_{20}+3\theta_{pi}^2 u_{02}-4\theta_{pi}u_{11}}\right]-\left[u_{20}+\theta_{si}^2 u_{02}-2\theta_{si}u_{11}\right]<0.$$

Similarly, efficiency conditions for presence of non-response as follows

Observation (A'):

$\boldsymbol{MSE_{min}}(\hat{\boldsymbol{T}}'_{\boldsymbol{pi}}) < \boldsymbol{MSE}(\hat{\boldsymbol{T}}'_{\boldsymbol{o}})$,

if

$$\left[1-\frac{\left(1+\theta_{pi}^2u_{02}-\theta_{pi}u_{11}\right)^2}{1+u_{20}+3\theta_{pi}^2u_{02}-4\theta_{pi}u_{11}}\right]-u_{20}<0,$$

Observation (B'):

$$\boldsymbol{MSE_{min}}\left(\boldsymbol{\hat{T}'_{pi}}\right)<\boldsymbol{MSE_{min}}\left(\boldsymbol{\hat{T}'_{se}}\right),$$

if

$$\left[1-\frac{\left(1+\theta_{pi}^2u_{02}-\theta_{pi}u_{11}\right)^2}{1+u_{20}+3\theta_{pi}^2u_{02}-4\theta_{pi}u_{11}}\right]-\left[\frac{u_{20}}{1+u_{20}}\right]<0,$$

Observation (C'):

$$\boldsymbol{MSE_{min}}\left(\boldsymbol{\hat{T}'_{pi}}\right)<\boldsymbol{MSE}\left(\boldsymbol{\hat{T}'_{isaki}}\right),$$

if

$$\left[1-\frac{\left(1+\theta_{pi}^2u_{02}-\theta_{pi}u_{11}\right)^2}{1+u_{20}+3\theta_{pi}^2u_{02}-4\theta_{pi}u_{11}}\right]-\left[u_{20}+u_{02}-2u_{11}\right]<0,$$

Observation (D'):

$$\boldsymbol{MSE_{min}}\left(\boldsymbol{\hat{T}'_{pi}}\right)<\boldsymbol{MSE}\left(\boldsymbol{\hat{T}'_{us}}\right),$$

if

$$\left[1-\frac{\left(1+\theta_{pi}^2u_{02}-\theta_{pi}u_{11}\right)^2}{1+u_{20}+3\theta_{pi}^2u_{02}-4\theta_{pi}u_{11}}\right]-\left[u_{20}+\theta_{us}^2u_{02}-2\theta_{us}u_{11}\right]<0,$$

Observation (E'):

$\boldsymbol{MSE_{min}}\left(\boldsymbol{\hat{T}'_{pi}}\right)<\boldsymbol{MSE}\left(\boldsymbol{\hat{T}'_{kci}}\right)$, for (i=1,2,3,4)

if

$$\left[1-\frac{\left(1+\theta_{pi}^2u_{02}-\theta_{pi}u_{11}\right)^2}{1+u_{20}+3\theta_{pi}^2u_{02}-4\theta_{pi}u_{11}}\right]-\left[u_{20}+\theta_{kci}^2u_{02}-2\theta_{kci}u_{11}\right]<0,$$

Observation (F'):

$\boldsymbol{MSE_{min}}\left(\boldsymbol{\hat{T}'_{pi}}\right)<\boldsymbol{MSE}\left(\boldsymbol{\hat{T}'_{si}}\right)$, for (i=1,2,3,4,5,6)

if

$$\left[1-\frac{\left(1+\theta_{pi}^2 u_{02}-\theta_{pi} u_{11}\right)^2}{1+u_{20}+3\theta_{pi}^2 u_{02}-4\theta_{pi} u_{11}}\right]-\left[u_{20}+\theta_{si}^2 u_{02}-2\theta_{si} u_{11}\right]<0.$$

## 6. NUMERICAL ILLUSTRATION

We use following data sets as follows:

**Population 1** We use the data set presented in Sarndal et al. (1992) concerning (P85) 1985 population in thousands considered as ($Y$) and (RMT85) revenues from 1985 municipal taxation (in millions of kronor), considered as ($X$). Descriptives of the population are $N$ = 234, $\bar{Y}$= 29.36268, $\bar{X}$ = 245.088, $S_y$ = 51.55674, $S_x$ = 596.3325, $\rho$ = 0.96, $\beta_2(y)$ = 89.23178, $\beta_2(x)$ = 89.18994, $\eta_{22}$= 4.0412, $Q_1$ = 67.75, $Q_2$ = 113.5, $Q_3$ = 230.25, $Q_r$ = 162.5, $Q_d$ = 81.25, $Q_a$ = 149 and $n$ = 35. We consider 20 % weight for non-response (missing values). So numerical results are provided only for 20 % missing values and considering last 47 values as non-respondents. Some important results from the population of non-respondents are as follows:

• For 20 %
$l$ = 2, $S_{y(2)}^2$ = 2.9167, $\beta_2(y_{(2)})$ = 11.7757, $N_2$ = 47.

**Population 2** We use the data set presented in Singh (2003) concerning ”Amount of real estate farm loans during (1977)” as ($Y$) and ”Amount of non-real estate farm loans during (1977) considered as ($X$). Descriptives of the population are $N$ = 50, $\bar{Y}$ = 555.4345, $\bar{X}$ = 878.1624, $S_y$ = 584.826, $S_x$ = 1084.678, $\rho$ = 0.80, $\beta_2(y)$ = 3.65531, $\beta_2(x)$ = 4.61704, $\eta_{22}$ = 2.8991, $Q_1$ = 63.4505, $Q_2$ = 452.517, $Q_3$ = 1177.151, $Q_r$ = 1113.7, $Q_d$ = 556.85, $Q_a$ = 620.30 and $n$ = 15. We consider 20 % weight for non-response (missing values). So numerical results are provided only for 20 % missing values and considering last 10 values as non-respondents. Some important results from the population of non-respondents are as follows:

• For 20 %
$l$ = 2, $S_{y(2)}^2$ = 244951.8, $\beta_2(y_{(2)})$ = 4.157837, $N_2$ = 10.

***Table 1.*** *PRE of reviewed and proposed estimators in absence of non-response*

| Estimator | PRE | Estimator | PRE | Estimator | PRE | Estimator | PRE |
|---|---|---|---|---|---|---|---|
| Pop-1 | | | | | | | |
| $\hat{T}_o$ | 100 | $\hat{T}_{kc2}$ | 434.63 | $\hat{T}_{s3}$ | 460.69 | $\hat{T}_{p2}$ | 872.87 |
| $\hat{T}_{se}$ | 321.02 | $\hat{T}_{kc3}$ | 432.74 | $\hat{T}_{s4}$ | 456.76 | $\hat{T}_{p3}$ | 656.42 |
| $\hat{T}_{isaki}$ | 434.74 | $\hat{T}_{kc4}$ | 434.69 | $\hat{T}_{s5}$ | 442.14 | $\hat{T}_{p4}$ | 777.85 |
| $\hat{T}_{us}$ | 434.84 | $\hat{T}_{s1}$ | 440.02 | $\hat{T}_{s6}$ | 454.57 | $\hat{T}_{p5}$ | 930.13 |
| $\hat{T}_{kc1}$ | 434.73 | $\hat{T}_{s2}$ | 447.99 | $\hat{T}_{p1}$ | 950.83 | $\hat{T}_{p6}$ | 804.06 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | Pop-2 | | | | |
| $\hat{\mathcal{T}}_o$ | 100 | $\hat{\mathcal{T}}_{kc2}$ | 118.01 | $\hat{\mathcal{T}}_{s3}$ | 155.06 | $\hat{\mathcal{T}}_{p2}$ | 174.51 |
| $\hat{\mathcal{T}}_{se}$ | 124.36 | $\hat{\mathcal{T}}_{kc3}$ | 118.16 | $\hat{\mathcal{T}}_{s4}$ | 156.15 | $\hat{\mathcal{T}}_{p3}$ | 187.01 |
| $\hat{\mathcal{T}}_{isaki}$ | 118.61 | $\hat{\mathcal{T}}_{kc4}$ | 118.37 | $\hat{\mathcal{T}}_{s5}$ | 153.02 | $\hat{\mathcal{T}}_{p4}$ | 188.74 |
| $\hat{\mathcal{T}}_{us}$ | 118.23 | $\hat{\mathcal{T}}_{s1}$ | 118.29 | $\hat{\mathcal{T}}_{s6}$ | 158.64 | $\hat{\mathcal{T}}_{p5}$ | 180.14 |
| $\hat{\mathcal{T}}_{kc1}$ | 118.13 | $\hat{\mathcal{T}}_{s2}$ | 143.34 | $\hat{\mathcal{T}}_{p1}$ | 180.33 | $\hat{\mathcal{T}}_{p6}$ | 183.33 |

***Table 2.*** *PRE of reviewed and proposed estimators in presence of non-response*

| Estimator | PRE | Estimator | PRE | Estimator | PRE | Estimator | PRE |
|---|---|---|---|---|---|---|---|
| | | | Pop-1 | | | | |
| $\hat{\mathcal{T}}'_o$ | 100 | $\hat{\mathcal{T}}'_{kc2}$ | 432.11 | $\hat{\mathcal{T}}'_{s3}$ | 457.85 | $\hat{\mathcal{T}}'_{p2}$ | 868.83 |
| $\hat{\mathcal{T}}'_{se}$ | 321.05 | $\hat{\mathcal{T}}'_{kc3}$ | 432.22 | $\hat{\mathcal{T}}'_{s4}$ | 453.97 | $\hat{\mathcal{T}}'_{p3}$ | 653.85 |
| $\hat{\mathcal{T}}'_{isaki}$ | 422.22 | $\hat{\mathcal{T}}'_{kc4}$ | 432.18 | $\hat{\mathcal{T}}'_{s5}$ | 439.53 | $\hat{\mathcal{T}}'_{p4}$ | 774.50 |
| $\hat{\mathcal{T}}'_{us}$ | 432.33 | $\hat{\mathcal{T}}'_{s1}$ | 437.44 | $\hat{\mathcal{T}}'_{s6}$ | 451.81 | $\hat{\mathcal{T}}'_{p5}$ | 925.63 |
| $\hat{\mathcal{T}}'_{kc1}$ | 432.22 | $\hat{\mathcal{T}}'_{s2}$ | 445.31 | $\hat{\mathcal{T}}'_{p1}$ | 946.16 | $\hat{\mathcal{T}}'_{p6}$ | 800.53 |
| | | | Pop-2 | | | | |
| $\hat{\mathcal{T}}'_o$ | 100 | $\hat{\mathcal{T}}'_{kc2}$ | 100.09 | $\hat{\mathcal{T}}'_{s3}$ | 126.28 | $\hat{\mathcal{T}}'_{p2}$ | 136.91 |
| $\hat{\mathcal{T}}'_{se}$ | 103.43 | $\hat{\mathcal{T}}'_{kc3}$ | 100.11 | $\hat{\mathcal{T}}'_{s4}$ | 126.85 | $\hat{\mathcal{T}}'_{p3}$ | 140.74 |
| $\hat{\mathcal{T}}'_{isaki}$ | 100.01 | $\hat{\mathcal{T}}'_{kc4}$ | 100.21 | $\hat{\mathcal{T}}'_{s5}$ | 116.41 | $\hat{\mathcal{T}}'_{p4}$ | 141.80 |
| $\hat{\mathcal{T}}'_{us}$ | 100.03 | $\hat{\mathcal{T}}'_{s1}$ | 101.33 | $\hat{\mathcal{T}}'_{s6}$ | 119.53 | $\hat{\mathcal{T}}'_{p5}$ | 139.52 |
| $\hat{\mathcal{T}}'_{kc1}$ | 100.07 | $\hat{\mathcal{T}}'_{s2}$ | 110.42 | $\hat{\mathcal{T}}'_{p1}$ | 129.90 | $\hat{\mathcal{T}}'_{p6}$ | 140.94 |

The percentage relative efficiencies of all the ratio type proposed and existing estimators available in Table 1 and 2. Note that

In absence of non-response PRE(.) $= \frac{MSE(\hat{\mathcal{T}}_o)}{MSE(.)}$,

In presence of non-response PRE(.) $= \frac{MSE(\hat{\mathcal{T}}'_o)}{MSE(.)}$.

## 7. CONCLUSION

In this article, we have suggested the ratio type class of estimators of finite population variance in absence and presence of non-response. The MSEs of the suggested ratio type class of estimators of finite population variance are obtained and compared with that of the usual unbiased estimator, ratio estimator, Upahyaya and Singh (1999) estimator, Kadilar and Cingi (2006) estimators and Solanki et al. (2015) estimators.

Further we have find the conditions for which the suggested estimators are more efficient than the reviewed estimators. On the premise of numerical illustration, we found that suggested estimators are better than the reviewed estimators for both situations for the considered populations.

## REFERENCES

[1] M. H. Hansen, and W. N. Hurwitz, The problem of non-response in sample surveys. Journal of the American Statistical Association, 41: 236 (1946) 517-529.

[2] C. T. Isaki, Variance estimation using auxiliary information. Journal of the American Statistical Association, 78: 381 (1983) 117-123.

[3] C. Kadilar and H. Cingi, 2006a. Ratio estimators for the population variance in simple and stratified random sampling. Applied Mathematics and Computation, 173: 2 (2006a) 1047-1059.

[4] C. Kadilar and H. Cingi, Improvement in variance estimation using auxiliary information. Hacettepe Journal of Mathematics and Statistics, 35:1 (2006b) 111-115.

[5] S. Riaz, G. Diana and J. Shabbir, A general class of estimators for the population mean using multiphase sampling with the non-respondents. Hacettepe Journal of Mathematics Statistics, 43:3 (2014) 511-527.

[6] C. E. Sarndal, B. Swensson and J. Wretman, Model Assisted Survey Sampling, New York:Springer, (1992).

[7] R. R. Sinha and V. Kumar, Families of estimators for finite population variance using auxiliary character under double sampling the non-respondents. National Academy Science Letters, 38:6 (2015) 501-505.

[8] J. Singh, B. N. Pandey and K. Hirano, On the utilization of a known coefficient of kurtosis in the estimation procedure of variance. Annals of the Institute of Statistical Mathematics, 25: 1 (1973) 51-55.

[9] H. P Singh, L. N. Upadhyaya and U. D Namjoshi, Estimation of finite population variance. Current Science, 57: 24 (1988) 1331-1334.

[10] B.K. Singh, W.W. Chanu and Y. Tato, A class of ratio cum dual to ratio estimator in presence of non-response using two phase sampling scheme. GU J Sci, 29:1 (2016) 213-221.

[11] R. S. Solanki, H. P. Singh and S. K. Pal, Improved ratio-type estimators of finite population variance using quartiles. Hacettepe J. Math. Stat, 44: 3 (2015.) 747-754.

[12] L. N. Upadhyaya, and H. P Singh, An estimator for population variance that utilizes the kurtosis of an auxiliary variable in sample surveys. Vikram Mathematical Journal, 19:1 (1999) 14-17.

[13] S. Singh, Advanced Sampling Theory with Applications: How Michael SelectedAmy. Kluwer Academic Publisher, The Netherlands, (2003) 1-1247.