New Operations over Generalized Interval Valued Intuitionistic Fuzzy Sets

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ABSTRACT

The concept of intuitionistic fuzzy sets and its generalizations play a vital role in modeling uncertainty and vagueness involved in different field of science. Recently, generalized interval valued intuitionistic fuzzy sets (GIVIFSBS) were presented by Baloui Jamkhaneh (2015) and he defined some operations over it. In this paper, defined arithmetic mean operation and geometric mean operation over GIVIFSBS were proposed and few theorems were proved. In addition, some of the basic properties of the new operations were discussed. By using these new operations, a prioritization method for generalized interval valued intuitionistic fuzzy judgment matrix was proposed.

Keywords: Generalized interval valued intuitionistic fuzzy sets, generalized intuitionistic fuzzy sets, arithmetic mean operation, geometric mean operation.

1. INTRODUCTION

After the introduction of fuzzy sets by Zadeh (1965), as an extension of crisp sets, a number of generalizations of this concept have come up. The intuitionistic fuzzy sets (IFSs) introduced by Atanassov (1986) is one among them. He defined some operations (as ∪, ∩, +, −) over IFSs. IFSs are a powerful tool to deal with vagueness and use it most comprehensive than fuzzy sets. Atanassov (1994) introduced operations @ and $ over the IFSs. Later on, researcher defined different operations over the IFSs. For example: De et al. (2000) Riecan and Atanassov (2006) Liu et al. (2008) Vasilev (2008), Riecan and Atanassov (2010) Parvathi et al. (2012) Wang and Liu (2013). Following the definition IFS, Atanassov and Gargov (1989) introduced interval valued intuitionistic fuzzy sets (IVIFSs) with several properties on IVIFSs and shown applications of IVIFSs. The membership and non-membership degrees of IVIFSs are intervals instead of real numbers, then it can contain more information. So IVIFSs are more powerful in dealing with vagueness and uncertainty than IFSs. Zhang et al. (2011) introduced a generalized interval valued intuitionistic fuzzy sets. Hui (2013) introduced some operations on interval valued intuitionistic fuzzy sets. Broumi and Smarandache (2014) introduced operations @ and $ over interval valued intuitionistic hesitant fuzzy sets due to Zhiming Zhang (2013) and studied several important properties. Zhao et al. (2016) studied derivative and differential operations on interval valued intuitionistic fuzzy.

Baloui Jamkhaneh (2015) considered a new generalized interval valued intuitionistic fuzzy sets (GIVIFSBS) and introduced some operators over GIVIFSBS. He studied the various basic operations like union, intersection, subset, complement and etc. All operations, defined over IVIFS were transformed for the GIVIFSBS case. In this paper, our aim is to propose two new operations @ and $ over GIVIFSBS and we will discuss their properties and propose a multi-criteria group decision making method based on the new operations.

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2. PRELIMINARIES

For completeness, some operators and necessity definitions on GIVIFSs are reviewed in this section.

Let $X$ be a non-empty set.

**Definition 2.1** (Atanassov, 1986) An IFS $A$ in $X$ is defined as an object of the form $A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$ where the functions $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ denote the degree of membership and degree of non-membership functions of $A$, respectively.

**Definition 2.2** (Atanassov & Gargov, 1989) Let $X$ be a non-empty set. Interval valued intuitionistic fuzzy sets (IVIFS) $A$ in $X$, is defined as an object of the form $A = \{(x, M_A(x), N_A(x)) : x \in X\}$ where $M_A(x) : X \rightarrow [0,1]$ and $N_A(x) : X \rightarrow [0,1]$ denote the degree of membership and degree of non-membership of $A$, respectively.

**Definition 2.3** (Baloui Jamkhaneh & Nadarajah, 2015) Let $X$ be a non-empty set. Generalized intuitionistic fuzzy sets (GIVIFS) $A$ in $X$, is defined as an object of the form $A = \{(x, M_A(x), N_A(x)) : x \in X\}$ where $M_A(x) : X \rightarrow [0,1]$ and $N_A(x) : X \rightarrow [0,1]$ denote the degree of membership and degree of non-membership functions of $A$, respectively.

**Definition 2.4** (Baloui Jamkhaneh, 2015) Let $X$ be a non-empty set. Generalized interval valued intuitionistic fuzzy sets (GIVIFS) $A$ in $X$, is defined as an object of the form $A = \{(x, M_A(x), N_A(x)) : x \in X\}$ where $M_A(x) : X \rightarrow [0,1]$ and $N_A(x) : X \rightarrow [0,1]$ denote the degree of membership and degree of non-membership of $A$, respectively.

**Definition 2.5** (Baloui Jamkhaneh, 2015) The degree of non-determinacy (uncertainty) of an element $x \in X$ to the GIVIFS $A$ is defined by

$$\pi_A(x) = [\pi_{M_A}(x), \pi_{N_A}(x)] = \left[ \left( 1 - M_{AU}(x) \right)^{\frac{1}{2}} - \left( 1 - N_{AU}(x) \right)^{\frac{1}{2}} \right].$$

**Definition 2.6** For every GIVIFS $A = \{(x, M_A(x), N_A(x)) : x \in X\}$, we define the modal logic operators “necessity” and “possibility”.

The Necessity measure on $A$:

$$\Box A = \left\{ (x, [M_{AL}(x), M_{AU}(x)], [(1 - M_{AU}(x))^{\frac{1}{2}}, (1 - M_{AL}(x))^{\frac{1}{2}}]) : x \in X \right\}.$$

The Possibility measure on $A$:

$$\Diamond A = \left\{ (x, \left[ (1 - N_{AL}(x))^{\frac{1}{2}}, (1 - N_{AL}(x))^{\frac{1}{2}} \right], [N_{AL}(x), N_{AU}(x)) : x \in X \right\}.$$

3. MAIN RESULTS

Here, we will introduce new operations over the GIVIFS $A$, which extend two operations in the research literature related to IVIFS. Let $X$ be a non-empty finite set.

**Definition 3.1** (Baloui Jamkhaneh & Gargov, 2015) For every GIVIFS as $A = \{(x, M_A(x), N_A(x)) : x \in X\}$ and $B = \{(x, M_B(x), N_B(x)) : x \in X\}$ we define the arithmetic mean operation and geometric mean operation as follows:

i. $A@B = \{(x, M_{AB}(x), N_{AB}(x)) : x \in X\}$

$$M_{AB}(x) = \left( \frac{1}{2} \left( M_{AL}(x)^{\delta} + M_{BL}(x)^{\delta} \right) \right)^{\frac{1}{\delta}},$$

$$N_{AB}(x) = \left( \frac{1}{2} \left( N_{AL}(x)^{\delta} + N_{BL}(x)^{\delta} \right) \right)^{\frac{1}{\delta}}.$$

ii. $A\ast B = \{(x, M_{AB}(x), N_{AB}(x)) : x \in X\}$

$$M_{AB}(x) = \left( \sqrt{M_{AL}(x) \cdot M_{BL}(x)} \right),$$

$$N_{AB}(x) = \left( \sqrt{N_{AL}(x) \cdot N_{BL}(x)} \right).$$

**Example 3.1** (Baloui Jamkhaneh & Gargov, 2015) Let $A = \{(x, [0.2, 0.3], [0.3, 0.4]) : x \in X\}$ and $B = \{(x, [0.3, 0.4], [0.2, 0.4]) : x \in X\}$.

Then the arithmetic mean operation and geometric mean operation as follows:

$$A@B = \{(x, \left[ \left( \frac{1}{2} \left( 0.2^2 + 0.3^2 \right) \right)^{\frac{1}{2}}, \left( \frac{1}{2} \left( 0.3^2 + 0.2^2 \right) \right)^{\frac{1}{2}} \right], \left( \frac{1}{2} \left( 0.2^2 + 0.4^2 \right) \right)^{\frac{1}{2}} : x \in X\}$$

$$A\ast B = \{(x, \left[ \left( \sqrt{0.2 \cdot 0.3}, \sqrt{0.3 \cdot 0.4} \right) \right], \left( \sqrt{0.3 \cdot 0.2, \sqrt{0.4 \cdot 0.4}} \right) : x \in X\}.$$
Theorem 3.1 For every two GIVIFSs $A$ and $B$, we have

1. $A @ B$ is a GIVIFS $B$.
2. $A B$ is a GIVIFS $B$.

Proof. (i) By using Definition 2.4, we have

\[ M_{A @ B}(x) = \frac{1}{a} (M_{A}(x) + M_{B}(x)) + \frac{1}{b} (N_{A}(x) + N_{B}(x)). \]

(ii) By using Definition 3.1, we have

\[ M_{A B}(x) = \frac{1}{a} (M_{A}(x) + M_{B}(x)) + \frac{1}{b} (N_{A}(x) + N_{B}(x)). \]

Finally, it can be concluded that $A @ B$ and $A B$ are GIVIFSs.

Theorem 3.2 For every two GIVIFSs $A$, $B$:

1. $A @ B = B A$.

Proof. (i) By using Definition 3.1, we have

\[ M_{A @ B}(x) = \left\{ \frac{1}{a} (M_{A}(x) + M_{B}(x)) \right\} C, \]

\[ M_{B A}(x) = \left\{ \frac{1}{b} (N_{A}(x) + N_{B}(x)) \right\} C. \]

The proof is completed. Proof (ii) is similar to that of (i).

Theorem 3.3 For every two GIVIFSs $A$, $B$:

1. $A @ A = A$.

Proof. These also follow from Definition 3.1.

Theorem 3.4 For every two GIVIFSs $A$, $B$:

(i) By using Definition 2.4, we have

\[ M_{A @ B}(x) = \frac{1}{a} (M_{A}(x) + M_{B}(x)) + \frac{1}{b} (N_{A}(x) + N_{B}(x)). \]

(ii) By using Definition 3.1, we have

\[ M_{A B}(x) = \frac{1}{a} (M_{A}(x) + M_{B}(x)) + \frac{1}{b} (N_{A}(x) + N_{B}(x)). \]

Therefore, $A @ B = A B$. The proof is completed. Proof (ii) is similar to that of (i).

Theorem 3.5 For every three GIVIFSs $A$, $B$, and $C$:

1. $(A @ B) @ C = (A @ C) @ (B @ C)$.
2. $(A B) @ C = (A @ C) @ (B @ C)$.

Proof. (i) By using Definition 3.1, we have

\[ M_{(A @ B) @ C}(x) = \left\{ \frac{1}{a} (M_{A}(x) + M_{B}(x)) \right\} C, \]

\[ M_{A @ (B @ C)}(x) = \left\{ \frac{1}{b} (N_{A}(x) + N_{B}(x)) \right\} C. \]

And
\[ N_{(A \cup B) \cap C}(x) = \left( \frac{1}{2} (N_{A \cap B}(x) + N_{C}(x)) \right)^{\frac{1}{2}}, \]
\[ \frac{1}{2} (N_{A \cap B}(x) + N_{C}(x))^{\frac{1}{2}}, \]
\[ = \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) \left[ \left( \frac{1}{2} \right) (N_{A}(x) + N_{B}(x))^{\frac{1}{2}} + N_{C}(x) \right]^{\frac{1}{2}}, \]
\[ = \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) (N_{A}(x) + N_{B}(x))^{\frac{1}{2}} + N_{C}(x) \right]^{\frac{1}{2}}. \]

Proof is complete. 

**Theorem 3.6** For every three GIVIFSs A, B, and C: (Distributive laws)

i. \( (A \cup B) \cap C = (A \cap C) \cup (B \cap C) \),

ii. \( (A \cap B) \cap C = (A \cap C) \cap (B \cap C) \),

iii. \( (A \cup B) \cap C = (A \cap C) \cup (B \cap C) \),

iv. \( (A \cap B) \cap C = (A \cap C) \cap (B \cap C) \).

**Proof** (i) Since \( A \cup B = \{ x, \max(M_a(x), M_B(x)), \min(N_a(x), N_B(x)) : x \in X \} \), we have

\[ M_{(A \cup B) \cap C} = \left[ \frac{1}{2} (\max(M_{A}(x), M_{B}(x)) + M_{C}(x)) \right]. \]

Proof is complete. Proof (ii) is similar to that of (i).

iii. \( M_{(A \cup B) \cap C} = \left[ \frac{1}{2} (\max(M_{A}(x), M_{B}(x)) + M_{C}(x)) \right]. \)
\[= \text{M}(\text{ASC}) \cup (\text{ASC})\].

And

\[N(\text{AUB}) = \left[\sqrt{\text{max}(N_{\text{AL}}(x), N_{\text{BL}}(x)), N_{\text{CL}}(x))}, \sqrt{\text{max}(N_{\text{AL}}(x), N_{\text{BL}}(x), N_{\text{CL}}(x))}\right].\]

\[= N(\text{ASC}) \cup (\text{ASC})\).

Proof is complete. Proof (iv) is similar to that of (iii).

**Theorem 3.7** For every two GIVIFSs A and B, we have (inclusion laws)

i. If \(A \subseteq B\) then \(A \cap B \subseteq B\).

ii. If \(A \cap B \subseteq B\).

**Proof.** (i) \(\text{if} a \leq b\) then \(a \leq \left(\frac{1}{\sqrt{a^2 + b^2}}\right)^\frac{1}{2} \leq b\). In this case, proof is clear.

(ii) \(\text{if} a \leq b\) then \(a \leq \sqrt{ab} \leq b\). In this case, proof is clear.

**Corollary 3.1** For every two GIVIFSs A and B, we have (Absorption laws)

i. \(A \cap (A \cup B) \subseteq A \cup B\).

ii. \(A \cup (A \cap B) \subseteq A \cup B\).

iii. \(A \cap (A \cap B) \subseteq A\).

iv. \(A \cup (A \cap B) \subseteq A\).

**Proof.** Since \(A \subseteq A \cup B\), \(A \cap B \subseteq A\), proofs are clear from Theorem 3.7.

**Theorem 3.8** For every three GIVIFSs A, B, and C:

i. \(\square(A \cap B) = \square A \cap \square B\).

ii. \(\square(A \cup B) = \square A \cup \square B\).

iii. \(\square A \cap B = \square A \cap \square B\).

iv. \(\square (A \cup B) = \square A \cup \square B\).

**Proof.** (i) By using Definition 3.1 and definition of Necessity measure, we have

\[\square(A \cap B) = \{x, M_{\text{cl}}(A \cap B), N_{\text{cl}}(A \cap B) : x \in X\},\]

since

\[M_{\text{cl}}(A \cap B) = \left[\left(\frac{1}{2} \left(M_{\text{AL}}(x)^6 + M_{\text{BL}}(x)^6\right)^\frac{1}{2}\right) + \left(M_{\text{CL}}(x)^6\right)^\frac{1}{2}\right] = M_{\text{cl}}(A \cap B),\]

\[N_{\text{cl}}(A \cap B) = \left[\left(1 - \left(\frac{1}{2} \left(M_{\text{AL}}(x)^6 + M_{\text{BL}}(x)^6\right)^\frac{1}{2}\right) + \left(M_{\text{CL}}(x)^6\right)^\frac{1}{2}\right)\right] = N_{\text{cl}}(A \cap B)\].

Therefore \(\square(A \cap B) = \square A \cap \square B\) is complete.

(ii) By using Definition 3.1 and definition of Necessity measure, we have

\[A \cap B = \{x, M_{\text{cl}}(A \cap B), N_{\text{cl}}(A \cap B) : x \in X\} =\]

\[\{x, \sqrt{M_{\text{AL}}(x)} \cdot M_{\text{BL}}(x) + M_{\text{AL}}(x) \cdot M_{\text{BL}}(x) : x \in X\},\]

To prove \(\square(A \cap B) \subseteq \square B \cap \square A\), it is enough to prove \(N_{\text{cl}}(A \cap B) \geq N_{\text{cl}}(A \cap B)\). It is clear

\[0 \leq a^6 + b^6 - 2(\sqrt{ab})^6\].

\[0 \leq a^6 + b^6 + (1 + (ab)^6) - (1 + (ab)^6) - 2(\sqrt{ab})^6\].

\[0 \leq (1 - a^6)(1 - b^6) \leq 1 + (ab)^6 - 2(\sqrt{ab})^6\].

\[0 \leq (1 - a^6)(1 - b^6) \leq 1 - (\sqrt{ab})^6\].

Proof is complete.

(iii) By using the definition of Possibility measure, we have

\[\diamond(A \cap B) = \{x, M_{\text{pl}}(A \cap B), N_{\text{pl}}(A \cap B) : x \in X\} = \]

\[\{x, \sqrt{1 - M_{\text{AL}}(x)^6} \cdot M_{\text{BL}}(x)^6 + (1 - M_{\text{AL}}(x)^6) \cdot M_{\text{BL}}(x)^6 : x \in X\},\]

Since

\[M_{\text{pl}}(A \cap B) = \left[\left(1 - \left(\frac{1}{2} \left(M_{\text{AL}}(x)^6 + M_{\text{BL}}(x)^6\right)^\frac{1}{2}\right) + \left(M_{\text{CL}}(x)^6\right)^\frac{1}{2}\right)\right] = N_{\text{pl}}(A \cap B)\].

\[= \left(\frac{1}{2} \left(M_{\text{AL}}(x)^6 + M_{\text{BL}}(x)^6\right)^\frac{1}{2}\right)\].
\[
\left\{ (1 - N_{AL}(x)^{\delta} ) + (1 - N_{BL}(x)^{\delta} ) \right\}^{\frac{1}{\delta}},
\]

\[= M_{\delta \star @B}.
\]

And

\[\emptyset A = \{(x, \left[ (1 - N_{AU}(x)^{\delta} ) \right]^{\frac{1}{\delta}}, \left[ (1 - N_{AL}(x)^{\delta} ) \right]^{\frac{1}{\delta}}, \left[ N_{AL}(x), N_{AU}(x) \right]\}, \forall x \in X.\]

Therefore \(\Box(A \oplus B) = \Box A \Box B.\)

Proof is complete. Proof (iv) are similar to that of (ii).

**Theorem 3.9** For every two GIVIFS\(\Box A\) and B. Distributive laws:

i. \(A \oplus B) = (A + C) @ (B + C),\)

ii. \((A \oplus B). C = (A . C) (B . C),\)

**Proof.** (i)

\[M_{(A \oplus B) + C} = \left\{ \frac{1}{\delta} \left( M_{AL}(x) \delta + M_{BL}(x) \delta \right) + M_{CL}(x) \delta - \frac{1}{\delta} \left( M_{AL}(x) \delta \right) \right. \]

\[+ M_{BL}(x) \delta M_{CL}(x) \delta \left( \frac{1}{\delta} \left( M_{AU}(x) \delta \right) \right) + M_{BU}(x) \delta + M_{CU}(x) \delta - \frac{1}{\delta} \left( M_{AU}(x) \delta \right) \]

\[+ M_{BU}(x) \delta M_{CU}(x) \delta \right\}. \]

Since

\[f_{(A \oplus B) + C} = \left\{ \frac{1}{\delta} \left( a \delta \right) \right\} + \left( b \delta \right) c \delta - \frac{1}{\delta} (a \delta) (b \delta), \]

\[= \left\{ \frac{1}{\delta} \left( a \delta + b \delta \right) \right\} + \left( b \delta \right) c \delta - \frac{1}{\delta} (a \delta) (b \delta), \]

\[= \left\{ \frac{1}{\delta} \left( a \delta + b \delta \right) + \left( b \delta \right) \right\} + \left( b \delta \right) c \delta - \frac{1}{\delta} \left( a \delta \right) \left( b \delta \right) c \delta, \]

\[= \left\{ \frac{1}{\delta} \left( a \delta + b \delta \right) \right\} + \left( b \delta \right) \right\} + \left( b \delta \right) c \delta - \frac{1}{\delta} \left( a \delta \right) \left( b \delta \right) c \delta = f_{(A \oplus C)(B \oplus C)}.
\]

Therefore \(M_{(A \oplus B) + C} = M_{(A + C) \oplus (B + C)},\)

\[N_{(A \oplus B) + C} = \left\{ \frac{1}{\delta} \left( N_{AL}(x) \delta \right) \right. \]

\[+ N_{BL}(x) \delta N_{CL}(x) \delta \left( \frac{1}{\delta} \left( N_{AU}(x) \delta \right) \right) \]

\[+ N_{BU}(x) \delta N_{CU}(x) \delta \right\}. \]

Since

\[g_{(A \oplus B) + C} = \left\{ \frac{1}{\delta} \left( a \delta + b \delta \right) \right\} c \delta = \left\{ \frac{1}{2} \left( a \delta c \delta + b \delta c \delta \right) \right\} = g_{(A \oplus C)(B \oplus C)}.
\]

Therefore \(N_{(A \oplus B) + C} = N_{(A + C) \oplus (B + C)}.\) Proof (ii) is similar to that of (i).

**Definition 3.2** For every GIVIFS\(\Box A\) as \(A_i =\)

\[\{(x, M_{A_i}(x), N_{A_i}(x)) : x \in X\}, \quad i = 1, ..., n\]

we define the arithmetic mean operation and geometric mean operation of \(A_i, \quad i = 1, ..., n\) as follows:

1. \(\oplus_{i=1}^n A_i = \{(x, M_{\oplus_{i=1}^n A_i}(x), N_{\oplus_{i=1}^n A_i}(x)) : x \in X\},\)

\[\quad M_{\oplus_{i=1}^n A_i}(x) = \left\{ \frac{1}{n} \left( \sum_{i=1}^n M_{A_i}(x) \right)^{\frac{1}{\delta}} \left( \sum_{i=1}^n N_{A_i}(x) \right)^{\frac{1}{\delta}} \right\}, \]

\[\quad N_{\oplus_{i=1}^n A_i}(x) = \left\{ \frac{1}{n} \left( \sum_{i=1}^n N_{A_i}(x) \right)^{\frac{1}{\delta}} \left( \sum_{i=1}^n M_{A_i}(x) \right)^{\frac{1}{\delta}} \right\} .\]

2. \(\ominus_{i=1}^n A_i = \{(x, M_{\ominus_{i=1}^n A_i}(x), N_{\ominus_{i=1}^n A_i}(x)) : x \in X\},\)

\[\quad M_{\ominus_{i=1}^n A_i}(x) = \left\{ \frac{1}{n} \left( \sum_{i=1}^n M_{A_i}(x) \right)^{\frac{1}{\delta}} \left( \sum_{i=1}^n N_{A_i}(x) \right)^{\frac{1}{\delta}} \right\}, \]

\[\quad N_{\ominus_{i=1}^n A_i}(x) = \left\{ \frac{1}{n} \left( \sum_{i=1}^n N_{A_i}(x) \right)^{\frac{1}{\delta}} \left( \sum_{i=1}^n M_{A_i}(x) \right)^{\frac{1}{\delta}} \right\} .\]

**Corollary 3.2** For every GIVIFS\(\Box A\) as \(A_i =\)

\[\{(x, M_{A_i}(x), N_{A_i}(x)) : x \in X\}, \quad i = 1, ..., n\]

we have

1. \(\oplus_{i=1}^n A_i \in \text{GIVIFS}_B,\)

2. \(\oplus_{i=1}^n k A_i = k \oplus_{i=1}^n A_i,\)

3. \(\oplus_{i=1}^n A_i = \oplus_{i=1}^n A_i,\)

4. \(\oplus_{i=1}^n A_i \oplus (A_k \oplus C) = \oplus_{i=1}^n (A_i \oplus C) .\)

**Corollary 3.3** For every GIVIFS\(\Box A\) as \(A_i =\)

\[\{(x, M_{A_i}(x), N_{A_i}(x)) : x \in X\}, \quad i = 1, ..., n\]

we have

1. \(\oplus_{i=1}^n A_i \in \text{GIVIFS}_B,\)

2. \(\oplus_{i=1}^n k A_i = k \oplus_{i=1}^n A_i,\)

3. \(\oplus_{i=1}^n A_i = \oplus_{i=1}^n A_i,\)

4. \(\oplus_{i=1}^n A_i \oplus (A_k \oplus C) = \oplus_{i=1}^n (A_i \oplus C) .\)

Using previous theorems simply proved to be these relations.

**4. PRIORITIZATION METHOD FOR GENERALIZED INTERVAL VALUED INTUITIONISTIC FUZZY JUDGMENT MATRIX**

In a multi-criteria decision making problem, suppose that there exists a set of criteria \(U = \{U_1, U_2, ..., U_n\}\). Preference \(U_i\) than \(U_j\) can be represented by a GIVIFS\(\Box A\) value as \(r_{ij} = \left( \{a_{ij}, b_{ij}\}, \{c_{ij}, d_{ij}\} \right)\), which can represent the membership degree and non-membership degree of the preference \(U_i\) than \(U_j\) for the interval.
valued intuitionistic fuzzy concept “excellence”. The decision makers provide intuitionistic fuzzy preference for each pair of criteria, and construct a generalized interval valued intuitionistic judgment matrix. A generalized interval valued intuitionistic fuzzy judgment matrix of $i^{th}$ decision maker is defined as follows.

$$r^{(i)} = \begin{bmatrix}
    r_{11}^{(i)} & r_{12}^{(i)} & \cdots & r_{1n}^{(i)} \\
    r_{21}^{(i)} & r_{22}^{(i)} & \cdots & r_{2n}^{(i)} \\
    \vdots & \vdots & \ddots & \vdots \\
    r_{n1}^{(i)} & r_{n2}^{(i)} & \cdots & r_{nn}^{(i)}
\end{bmatrix},$$

where $r_{ij}^{(i)} = \left( M_{ij}^{(i)}, N_{ij}^{(i)} \right)$, $M_{ij}^{(i)} = \left( M_{ij}^{(i)} \right)^{\delta}$, $M_{ij}^{(i)} = \left( N_{ij}^{(i)} \right)^{\delta}$, $N_{ij}^{(i)} \leq 1$. $N_{ij}^{(i)} = N_{ij}^{(i)}$, $M_{ij}^{(i)} = N_{ij}^{(i)}$. $r_{ii}^{(i)} = \left( 0, 0 \right)$, $i = 1, \ldots, n, j = 1, \ldots, n, l = 1, \ldots, k$.

The next four steps can summarize the criteria ranking.

Step1. Calculate the finally judgment matrix $r = \left( r_{ij} \right)_{n \times n}$ using combine interval-valued intuitionistic fuzzy judgment matrix of decision makers, $r_{ij} = \left( \sum_{k=1}^{n} s_{ij}^{k} s_{kl}^{k}, \sum_{k=1}^{n} s_{ij}^{k} N_{kl}^{(k)} \right)$, $i = 1, \ldots, n$, $j = 1, \ldots, n$.

Step2. The collective overall preference values of $i^{th}$ criteria calculate as follows

$$r_{i} = \left( \sum_{j=1}^{n} r_{ij} s_{ij}^{k}, \sum_{j=1}^{n} r_{ij} N_{ij}^{(k)} \right), i = 1, \ldots, n.$$

Step3. Calculate score function $S(r_{i})$ and accuracy function $H(r_{i})$

$$\Delta = \left( \sum_{j=1}^{n} r_{ij} s_{ij}^{k}, \sum_{j=1}^{n} r_{ij} N_{ij}^{(k)} \right) - \left( \sum_{j=1}^{n} s_{ij}^{k}, \sum_{j=1}^{n} N_{ij}^{(k)} \right),$$

$$S(r_{i}) = \frac{\Delta^{2} \text{sgn}(\Delta)}{2},$$

$$H(r_{i}) = \frac{\left( \sum_{j=1}^{n} r_{ij} s_{ij}^{k}, \sum_{j=1}^{n} r_{ij} N_{ij}^{(k)} \right)^{2} + \left( \sum_{j=1}^{n} s_{ij}^{k}, \sum_{j=1}^{n} N_{ij}^{(k)} \right)^{2}}{2}.$$

Step4. Ranking order of the criteria is as follows

(i) if $S(r_{i}) < S(r_{j})$ then $r_{i}$ is smaller than $r_{j}$, denoted by $r_{i} < r_{j}$, that is, $U_{ij}$ has higher priority than $U_{ji}$.

(ii) if $S(r_{i}) > S(r_{j})$ then $r_{i}$ is bigger than $r_{j}$, denoted by $r_{i} > r_{j}$, that is, $U_{ji}$ has higher priority than $U_{ij}$.

(iii) if $S(r_{i}) = S(r_{j})$ and $H(r_{i}) < H(r_{j})$ then $r_{i}$ is smaller than $r_{j}$.

(iv) if $S(r_{i}) = S(r_{j})$ and $H(r_{i}) > H(r_{j})$ then $r_{i}$ is bigger than $r_{j}$.

(v) if $S(r_{i}) = S(r_{j})$ and $H(r_{i}) = H(r_{j})$ then $r_{i}$ and $r_{j}$ represent the same information, denoted by $r_{i} = r_{j}$, that is, priority $r_{i}$ and $r_{j}$ are same.

Example 4.1 A customer intends to buy a car. The customer takes into account the following three criteria: design ($S_1$), price ($S_2$), level of after-sale service ($S_3$). Generalized interval-valued intuitionistic fuzzy judgment matrix of decision makers ($\delta = 2$) are as follows.

$$r^{(1)} = \begin{bmatrix}
    (0.5, 0.5), (0.3, 0.6) & (0.1, 0.3), (0.5, 0.7), (0.1, 0.2) \\
    (0.2, 0.2), (0.5, 0.5) & (0.4, 0.5), (0.2, 0.3) \\
    (0.1, 0.2), (0.6, 0.7) & (0.2, 0.4), (0.5, 0.7), (0.5, 0.5)
\end{bmatrix}.$$

The finally judgment matrix is as follows

$$r = \begin{bmatrix}
    (0.5, 0.5), (0.3, 0.6), (0.1, 0.3), (0.5, 0.7), (0.1, 0.2) \\
    (0.2, 0.2), (0.5, 0.5), (0.4, 0.5), (0.2, 0.3) \\
    (0.1, 0.2), (0.6, 0.7), (0.2, 0.4), (0.5, 0.7), (0.5, 0.5)
\end{bmatrix}.$$

The collective overall preference values of criteria and their score function are as follows

$$r_1 = [0.450, 0.588, 0.303, 0.341, 0.328, 0.2927],$$

$$r_2 = [0.391, 0.468, 0.341, 0.4722],$$

$$S(r_2) = 0.0914,$$

$$r_3 = [0.316, 0.369, 0.499, 0.6026].$$

Therefore, the ranking order of the three criteria is $S_1$, $S_2$ and $S_3$.

5. Conclusions

In this paper, we have defined two new operations over generalized interval valued intuitionistic fuzzy sets and their relationships are proved. We have studied some desirable properties of the proposed operations, such as idempotent laws, complementary law, commutative laws, distributive laws and etc. and applied the these operations to decision making with generalized interval valued intuitionistic fuzzy information.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.
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